RESEARCH DEPARTMENT

Ray tracing for ionospheric and tropospheric propagation studies

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THE BRITISH BROADCASTING CORPORATION
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RESEARCH DEPARTMENT

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RAY TRACING FOR IONOSPHERIC AND TROPOSPHERIC PROPAGATION STUDIES

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RAY TRACING FOR IONOSPHERIC AND TROPOSPHERIC PROPAGATION STUDIES

SUMMARY

Adapted for a digital computer, conventional ray tracing becomes a potentially useful technique for studying several aspects of long-range wave-propagation through stratified media. The development of a ray-tracing programme and its application to both medium-frequency sky-wave propagation through the ionosphere and ultra-high-frequency (u.h.f.) propagation in the troposphere are described in this report.

1. INTRODUCTION

In propagation through inhomogenous or stratified media, methods based on a rigorous wave-theory solution are usually cumbersome and their practical application lengthy, even if it can be assumed that the model representing the medium is accurate and realistic enough to justify such an elaborate treatment. On the other hand, a geometrical-optics approach, although only an approximation, is relatively simple in conception and application. In fact, it has been shown in many instances to lead to a surprisingly good estimate of the general behaviour of propagated waves. Furthermore, ray tracing based on geometrical-optics principles is readily adapted to evaluation by computer, because of its simple iterative nature.

It is not intended to discuss here the degree of validity of a ray-theory approach to long-range propagation. We assume from the outset that the variations in the refractive index of the medium with height are sufficiently slow compared with the wavelength and, in the application to long-range tropospheric propagation that the atmospheric duct guiding the radiation is sufficiently extensive, horizontally and vertically. Even so, the ray-tracing interpretation of the well-known ducting phenomenon in the troposphere (at u.h.f. for example) can produce a complicated pattern of rays indicating the possibility of strong fields at certain points in the propagation range. (An example of this can be seen in Fig. 5.) This interpretation cannot readily give quantitative results when strong ducting is present but can indicate whether a certain refractive index profile would be expected to favour enhanced fields at great distances.

For application to ionospheric propagation, the method developed is expected to be valid for both the medium-frequency and high-frequency broadcast bands. At frequencies less than the gyromagnetic frequency (approximately 1 MHz) the method should not be used for waves launched with angles of elevation greater than about 30°. This restriction can be relaxed for high-frequency propagation.

One application of ray tracing at medium frequencies is the determination of absorption, and it may be important in some applications (e.g. in the calculation of ionospheric cross-modulation) to know not only the total absorption for the path but also the relative amounts of absorption that occur at different points along the path. The method of calculation is therefore extended to include absorption, as discussed in Section 4.3.

The method is based on the well established principle that the wave-normal obeys Snell's law of refraction and on the assumption that the propagation characteristics of the medium are spherically stratified, concentric with a spherical Earth. Snell's law, applied to spherically curved layers takes the form (Fig. 1).

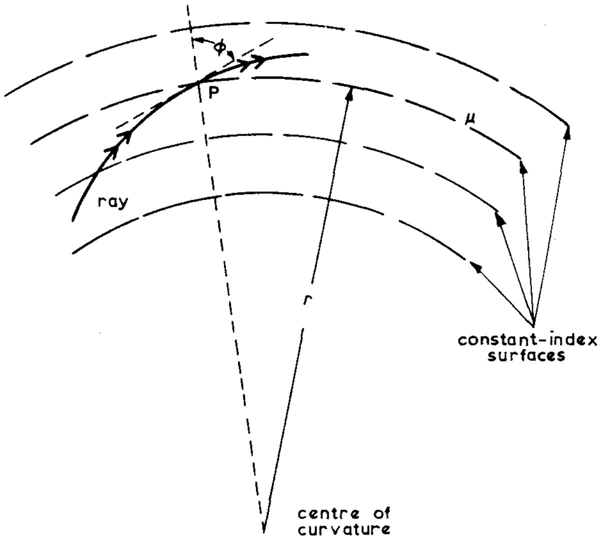

Fig. 1 - Snell's law applied to a spherically stratified medium

$$\mu r \sin \phi = \text{constant} \tag{1}$$

where, at a given point in the medium, P in Fig. 1 say,

μ = the real part of the refractive index of the medium at P

r = the radius of curvature of the constant-index sphere through P

and ϕ = the angle between the wave-normal and the normal to the constant-index sphere at P, i.e. the angle of incidence at P.

Clearly, because of the initial assumptions, r will be a linear function of the vertical height h of the ray above the ground and the refractive index will be a continuous function of h. Thus, by virtue of Equation (1), the direction of the ray will be a function of h, and if this is computed for a sufficient number of points the ray trajectory is established.

The application to tropospheric propagation at u.h.f. is simplified by the fact that the refractive index is everywhere real and independent of the Earth's magnetic field, since the numbers of free electrons and gas ions are insignificant. Here, μ is a function of h only.

Waves penetrating into the magneto-ionic plasma associated with the ionosphere, however, are influenced by the Earth's magnetic field in a complicated manner[1,2]. The incident wave is split into two components, the ordinary and extraordinary waves, which travel generally in different directions, and energy is absorbed from each of them by the medium. Furthermore, at any point the wave energy in each component is propagated in a slightly different direction to the direction of the wave-normal and is not confined to the initial plane of incidence. Although the latter effect can be allowed for in the ray-tracing calculations, it is neglected in the treatment described below because, for the applications of the method envisaged, it is believed that the resulting error will be small.

The complex refractive index of the magneto-ionic medium for the ordinary and extraordinary components is taken to be that predicted by the Appleton-Hartree formulae.* These formulae show that the real part of the complex refractive index is a function of several variables, one of which is the angle between the ray direction and that of the Earth's magnetic field. This fact complicates the situation somewhat because Snell's law (Equation (1)) cannot then be solved explicitly. In the step-by-step procedure outlined below, however, this difficulty can be largely overcome.

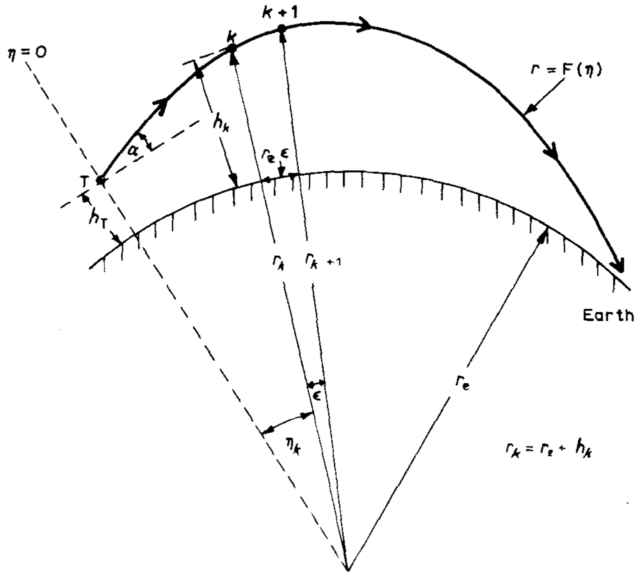

Fig. 2 - The ray trajectory: co-ordinates and notation

2. THE RAY TRAJECTORY

Fig. 2 shows the trajectory of a typical ray launched from a point near the Earth's surface with an angle of elevation α (take-off angle). Using polar co-ordinates (r, η) with the origin at the centre of the Earth and the diameter through the transmitter ($\eta = 0$) for angular reference, the trajectory (which lies in a great circle plane) is described by $r = F(\eta)$. Let r_e be the radius of the Earth and h the vertical height of a point on the ray above the ground, so that $r = r_e + h = F(\eta)$.

Consecutive points on the trajectory which are separated by a small angular interval, ϵ say, are expected to be well-related by the first few terms of an infinite Taylor series. The series relation is:

$$F(\eta + \epsilon) = F(\eta) + \epsilon\frac{dF}{d\eta} + \frac{\epsilon^2}{2}\frac{d^2F}{d\eta^2} + \frac{\epsilon^3}{6}\frac{d^3F}{d\eta^3} + \ldots \tag{2}$$

Since the trajectories pertaining to wave-propagation problems are usually reasonably smooth and well-behaved, it is estimated that for a ground range interval $\Delta (= r_e\epsilon$ see Fig. 2) of a few kilometres the first three terms of the Taylor series will suffice.

Let r_{k+1} be the height the ray $r = F(\eta)$ has reached after the $(k + 1)^{th}$ interval; then from Equation (2)

$$r_{k+1} \approx r_k + \epsilon\left(\frac{dr}{d\eta}\right)_k + \frac{\epsilon^2}{2}\left(\frac{d^2r}{d\eta^2}\right)_k \tag{3}$$

If μ_0, r_0 and ϕ_0 are the initial values of these variables at the beginning of the trajectory we have from Equation (1), $\mu r \sin \phi = \mu_0 r_0 \sin \phi_0 =$ constant.

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* These formulae are fully discussed in Ref. 1.

Making the substitution $\sin\phi_o = \cos\alpha$ and rearranging gives

$$\operatorname{cosec}\phi = \left(\frac{\mu}{\mu_o}\right)\left(\frac{r}{r_o}\right)\sec\alpha \qquad (4)$$

where (Fig. 2)

α = the take-off angle

r_o = the value of r for the transmitting point

μ_o = the refractive index at $r = r_o$.

Now it may be shown that

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\eta}\right)_k = r_k(\operatorname{cosec}^2\phi_k - 1)^{1/2} \qquad (5)$$

and, by differentiating Equation (1) to obtain $(\mathrm{d}\phi/\mathrm{d}r)$, that

$$\left(\frac{\mathrm{d}^2r}{\mathrm{d}\eta^2}\right)_k = r_k\left\{\left[2 + \frac{r_k}{\mu_k}\left(\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_k\right]\operatorname{cosec}^2\phi_k - 1\right\} \qquad (6)$$

whereupon, from Equations (4), (5) and (6), Equation (3) becomes

$$\begin{aligned} r_{k+1} \approx r_k + r_k\epsilon\left[\left(\frac{r_k}{r_o}\right)^2\left(\frac{\mu_k}{\mu_o}\right)^2\sec^2\alpha - 1\right]^{1/2} + \\ + r_k\frac{\epsilon^2}{2}\left\{\left[2 + \left(\frac{r_k}{\mu_k}\right)\left(\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_k\right]\left(\frac{r_k}{r_o}\right)^2 \times \right. \qquad (7) \\ \left. \times\left(\frac{\mu_k}{\mu_o}\right)^2\sec^2\alpha - 1\right\} \end{aligned}$$

Equation (7) is the basic iterative relation which is used to compute the ray trajectory. Below, further reductions of Equation (7) are developed which are better adapted to the characteristics of the variables in the particular application.

3. LONG-RANGE U.H.F. PROPAGATION IN THE TROPOSPHERE

3.1. General Formulation

It is well known that one mechanism for long-range propagation at u.h.f. is the ducting or guiding of radiation by an abnormal combination of refractive-index gradients in the lower regions of the troposphere. The conditions which are conducive to such propagation phenomena are readily explored by ray-tracing techniques. It must be pointed out, however, that if the atmospheric duct is regarded as a waveguide, the possibility exists of the latter being beyond cut-off for the wavelength considered if the effective height of the guide is too small. This possibility is not indicated by a ray-tracing approach which must, therefore, be used with some caution in this application.

The refractive index gradients of the air layers are very weak so that propagation much beyond the normal horizon occurs only for waves launched with very small take-off angles, say $-1° < \alpha < 1°$, and in Equation (7) $\sec^2\alpha$ can be replaced by $1 + \alpha^2$. For similar reasons the maximum height of any significant ray is not likely to exceed 2 km above the ground. The refractive index, μ, of the air in the troposphere is always near unity and, therefore, it is more convenient to specify the index in N-units, i.e., as $N \equiv (\mu - 1)10^6$.

Bearing in mind these limitations on the range of the variables, Equation (7) can be reduced to read

$$\begin{aligned} h_{k+1} \approx h_k \pm\Delta\left|\left[0{\cdot}0846\alpha^2 + 0{\cdot}314\left(h_k - h_T\right) + 2\left(N_k - N_T\right)\right]^{1/2}\right| + \\ + 0{\cdot}0785\Delta^2\left[1 + 6{\cdot}37\left(\frac{\mathrm{d}N}{\mathrm{d}h}\right)_k\right] \qquad (8) \end{aligned}$$

where

h_k = height of ray above the ground in metres at the ground range $k\Delta$, k = 1, 2, 3, etc.

h_T = height of transmitting aerial in metres

N_k = refractive index at the height h_k in N-units

N_T = refractive index at the height h_T in N-units

α = take-off angle in minutes of arc

$\left(\frac{\mathrm{d}N}{\mathrm{d}h}\right)_k$ = gradient of the refractive-index at the height h_k (N-units/metre).

Δ = selected ground-range interval in kilometres.

The Earth's radius* is taken to be $6{\cdot}37 \times 10^6$ metres.

3.2. Sign Determination

Measurements of the refractive index of air often reveal a very irregular distribution with height and the gradient $\mathrm{d}N/\mathrm{d}h$ may be positive or negative, although usually the latter. It will be seen from Equation (8) that the sign of the Δ^2 term is uniquely determined by the magnitude and sign of the gradient $\mathrm{d}N/\mathrm{d}h$ and, therefore, by the input data. The second term in Equation (8), however, contains a square-root factor and whether the positive or negative root is taken depends on whether the ray height is increasing or decreasing respectively. The ray-tracing programme must contain instructions to determine the correct sign for this term at each step in the calculation.

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* Note that a smooth, spherical Earth was assumed at the outset.

The following procedure is found to be satisfactory:

(a) Initial step: choose the positive sign if α is positive and the negative sign if α is negative. If $\alpha = 0$ choose the same sign as the Δ^2 term.

(b) Subsequent steps: Form $h_{k+1} - h_k$ using the sign determined by (a), and change the sign whenever $|h_{k+1} - h_k| \leqslant |\Delta^2 \text{ term}|$. (This rule determines when the ray height has passed through a maximum or a minimum.)

3.3. Input Data

The refractive-index profile is presented as a sequence of height-index pairs of values (height in metres, index in N-units). The height values are not in general equispaced but are given in increasing order. The index N_k corresponding to the height h_k is obtained by first-order interpolation of the given pair-values. The gradient $(\mathrm{d}N/\mathrm{d}h)_k$ is assumed to be constant between successive pairs of values: thus if h_k lies between h_j and h_{j+1} the gradient is taken to be $(N_{j+1} - N_j)/(h_{j+1} - h_j)$.

The range interval Δ, the transmitting-aerial height, and the take-off angles required constitute the rest of the input data.

3.4. Output Data

The data printed out depends largely on the form of the ray-trace investigation. A useful exploratory arrangement is to compute the trajectory for each of a fan of rays with take-off angles centred on $\alpha = 0$, e.g. $-0°10'$ to $+0°10'$ in $0{\cdot}5'$ steps. The calculation is stopped when the ray either hits the earth or 'escapes', i.e. when the ray height exceeds the maximum height of the profile input data. The ray height at each step (or at intervals of several steps) is printed out if a detailed study of the trajectory is required. On other occasions, printing out only those portions of the trajectory near the turning points will suffice.

Crude ray-focusing occurs with some index profiles and an estimate of the convergence gain in these caustic regions can be obtained by printing out the ray heights, for each of the fan of rays, at a specific ground range. The number of rays per unit height relative to that expected under free-space conditions (straight-line rays) gives a measure of the power gain.

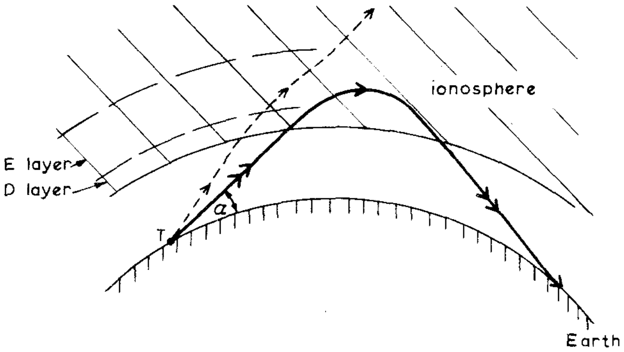

Fig. 3 - Oblique ionospheric propagation

4. M.F. AND H.F. SKY-WAVE PROPAGATION VIA THE IONOSPHERE

In this problem straight-line propagation through the tropospheric region is assumed but refraction occurs on entering the ionospheric region. Waves entering at sufficiently oblique angles of incidence (Fig. 3) are deviated back to Earth in the E layer after passing through the D layer, giving rise to long-range propagation. Further, wave energy is lost to the magneto-ionic medium and this absorption loss may differ considerably in magnitude for the ordinary and extraordinary waves respectively. Both the ray trajectory and the absorption loss can be determined, for each component, if the variation of the complex refractive index with height is known. Each component is treated separately and it is assumed that an ordinary wave remains ordinary while traversing the ionosphere. This is justified in the absence of large changes of refractive index within height intervals comparable with a wavelength. If sudden changes are present, propagation will involve partial reflections and simple ray-tracing techniques will not be accurate.

The calculations do not include polarization coupling losses, i.e. do not allow for the fraction of the power in the wave radiated from the transmitting aerial (usually linearly polarized) that excites the ordinary or extraordinary mode on entering the ionosphere.

4.1. Complex Refractive Index

The Appleton-Hartree formula for the complex refractive index $(\mu - \mathrm{j}\chi)$ is given by

$$(\mu-\mathrm{j}\chi)^2 = 1-X\left\{1-\mathrm{j}Z-\frac{Y^2\sin^2\theta}{2(1-X-\mathrm{j}Z)} \pm \left[\frac{Y^4\sin^4\theta}{4(1-X-\mathrm{j}Z)^2}+Y^2\cos^2\theta\right]^{1/2}\right\}^{-1} \tag{9}$$

where μ = real part of the refractive index

χ = imaginary part of the refractive index. (N.B. $\chi = 0$ if $Z = 0$ or $X = 0$).

$X = 81\ E/f^2$; E is the free-electron density in number/cm^3 and f is the wave frequency in kHz.

Y = Gyromagnetic frequency divided by the wave frequency.

θ = angle between geomagnetic field and ray direction.

$2\pi Z$ = collision frequency divided by the wave frequency.

($\pm$) = + sign for ordinary wave, − sign for extraordinary wave.

The angle θ in Equation (9) is calculated from the trigonometric formula

$$\cos\theta = \cos\psi\ \sin\phi\ \cos\gamma - \sin\psi\ \cos\phi \qquad (10)$$

where ψ = angle of magnetic dip: range −90° to +90° (positive in northern latitudes). N.B. the dip angle may vary significantly even over that part of the trajectory which is through the ionosphere. Usually, however, it is sufficient to take its value at the mid-point of the path.

ϕ = angle between the upwards vertical and the ray direction: range 0° to 180°.

γ = ray magnetic bearing: range 0° to 360° east of magnetic north. N.B. In addition to variations due to changes of magnetic declination along the path the ray bearing will be subject to a systematic variation but, except possibly for very high latitudes, variations from the initial value may be neglected in the present application.

The variations of electron density and collision frequency with height form the main input data, together with the other necessary physical constants. It should be noted that the angle ϕ, which is required to evaluate Equation (10), is a function of the ray direction (see Equation (1)) and cannot therefore be obtained directly from the input data.

4.2. Calculation Procedure

As in the previous (tropospheric) application, a ground range interval is selected and the ray height calculated at the end of each interval using the basic iterative relation of Equation (7). It is convenient to re-state the latter in the form

$$p_{k+1} \approx p_k + p_k\epsilon\left[p_k^2\mu_k^2\sec^2\alpha - 1\right]^{1/2} + p_k\frac{\epsilon^2}{2}\left\{\left[2 + \frac{p_k}{\mu_k}\left(\frac{d\mu}{dp}\right)_k\right]p_k^2\mu_k^2\sec^2\alpha - 1\right\} \qquad (11)$$

where p_k = relative height $(r_k/r_e) = 1 + h_k r_e$ ($r_e = 6{\cdot}37 \times 10^6$ metres)

ϵ = angular range interval (range corresponding to height p_k is $kr_e\epsilon$).

μ_k = real part of refractive index at the relative height p_k.

α = take-off angle (transmitting aerial assumed at ground level).

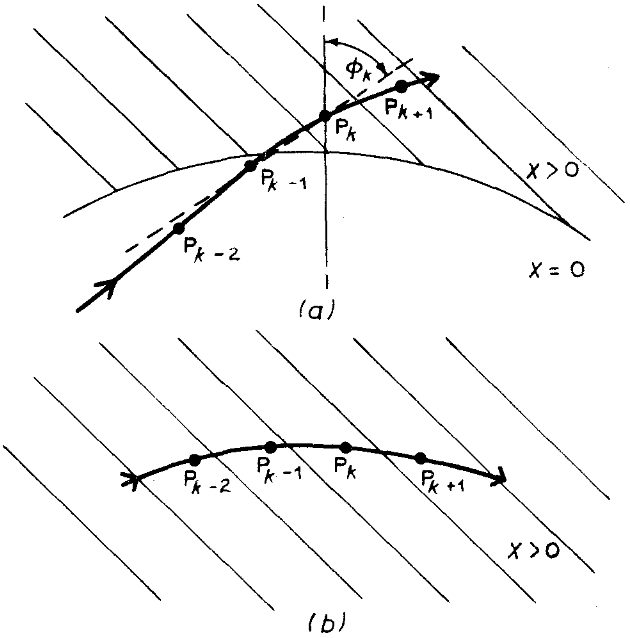

Fig. 4 - Calculation procedure
(a) points at entry into ionosphere
(b) points at the turn-over

Consider (Fig. 4(a)) points on that portion of the trajectory where it enters the ionosphere, i.e. at the height where X in Equation (9) first becomes significantly different from zero. Up to this height the refractive index is assumed real and equal to unity so that no absorption or refraction has occurred. In Fig. 4(a), p_k marks the first step into the ionosphere and μ_k and χ_k are calculated using Equations (9) and (10) with the approximation

$$\phi_k \approx \arctan\left(\frac{\epsilon}{p_k - p_{k-1}}\right) \qquad (12)$$

Next, the index gradient $(d\mu/dp)_k$ is obtained from the approximation

$$\left(\frac{d\mu}{dp}\right)_k \approx \frac{\mu_k - \mu_{k-1}}{p_k - p_{k-1}} \qquad (13)$$

Finally, p_{k+1} is calculated from Equation (11) and the procedure repeated for subsequent steps.*

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* The sign of the square root factor in Equation 11 is determined by the rules given in Section 3.2.

It will be seen that the approximation in Equation (13) can become rather indeterminate at the turn-over point of the trajectory (Fig. 4(*b*)) since both numerator and denominator may then approach zero. Furthermore, at this point, ϕ_k passes through $\pi/2$ and the second term on the right-hand side of Equation (11) vanishes, so that the third term containing the index gradient $(d\mu/dp)_k$ is here very significant. In order to avoid a possible distortion of the ray trajectory at the turnover the index gradient is calculated from the partial derivative of the Appleton-Hartree formula, with respect to the relative height p, whenever $|p_k-p_{k-1}|<g$, where g is an arbitrarily chosen small quantity. The coefficient $(d\mu/dp)_k$ in Equation (11) is then replaced by $(\partial\mu/\partial p)_k$, the formula for which is given in the Appendix.

4.3. Absorption

An important part of the application to m.f. propagation is the ability to assess the absorption losses and the way these vary with height, range and frequency. As previously mentioned the imaginary part χ, of the complex refractive index, is calculated at the end of each interval simultaneously with the real part μ.

Let χ_{k-1} and χ_k be successively computed values of the imaginary part. The average value is $(\chi_{k-1}+\chi_k)/2$ and it may be shown that the average rate of energy loss over this interval is

$$0{\cdot}09083\, f\, (\chi_{k-1}+\chi_k) \text{ decibels/km}$$

(f = wave frequency in kHz)

The length of the interval along the ray trajectory is approximately

$$r_e\left[\left(p_k-p_{k-1}\right)^2+p_{k-1}^2\epsilon^2\right]^{\frac{1}{2}}$$

so that the absorption loss, $L_{k,k-1}$ say, is given by

$$L_{k,k-1}\approx 0{\cdot}09083\, fr_e\, (\chi_{k-1}+\chi_k)\left[(p_k-p_{k-1})^2+p_{k-1}^2\epsilon^2\right]^{\frac{1}{2}} \text{ decibels}$$

In the ray-tracing programme the running-total absorption loss

$$\sum_1^k L_{k,k-1}$$

is calculated and printed out together with the range and ray height at the end of each interval.

4.4. Input Data

The electron-density and collision-frequency profiles of the model ionosphere are entered as a sequence of values at intervals of one kilometre, normally extending from about 60 km to 110 km for medium-frequency propagation.* A programme subroutine carries out a first-order interpolation of these data to obtain the values at intermediate heights. A ray is deemed to have 'escaped' and further calculation is stopped if the calculated ray height exceeds the highest value entered for the profile.

Other essential input data comprise the take-off angle, ray bearing, magnetic-dip angle, gyro-frequency, wave frequency and range interval. Separate calculations are necessary for the ordinary and extraordinary rays and the required case is selected by entering the reference number +1 or −1 respectively.

4.5. Output Data

The range, ray height and cumulative absorption loss are printed-out for each step in the ionosphere. If the ray returns to earth the total range is calculated and printed-out, otherwise 'ray escapes' is printed.

4.6. Additional Programme Features

One application of ray tracing is to estimate the ionospheric absorption loss for propagation over a specific link. In this case the geographical locations of the transmitter and receiver are known and interest centres on ray traces within a narrow range of take-off angles. A sub-routine has been included in the main programme to deal with this requirement.

The latitude and longitude of both terminal points are entered as input data from which the total range and magnetic bearing required are calculated, using straightforward trigonometric formulae. Further, again using standard formulae, an estimate is made of the nominal take-off angle required (a typical virtual height of about 100 km is assumed for E-layer reflection). Three ray traces are then computed for the take-off angles $(\alpha_j-1)°$, $\alpha_j°$ $(\alpha_j+1)°$ where α_j is the nominal angle required.

The corresponding ground ranges so obtained are generally found to straddle the exact range required and, by subsequent graphical interpolation or inspection, it is easy to see how the absorption loss behaves in the region of interest. Also, from the increments of ground range between the successive rays, the actual convergence gain can be deduced. However, it has been found that this differs little from the convergence gain produced by a spherical mirror at the virtual height.

* For high-frequency applications, where the reflection heights are much greater, the height intervals at which data are entered and the upper limit can be increased accordingly.

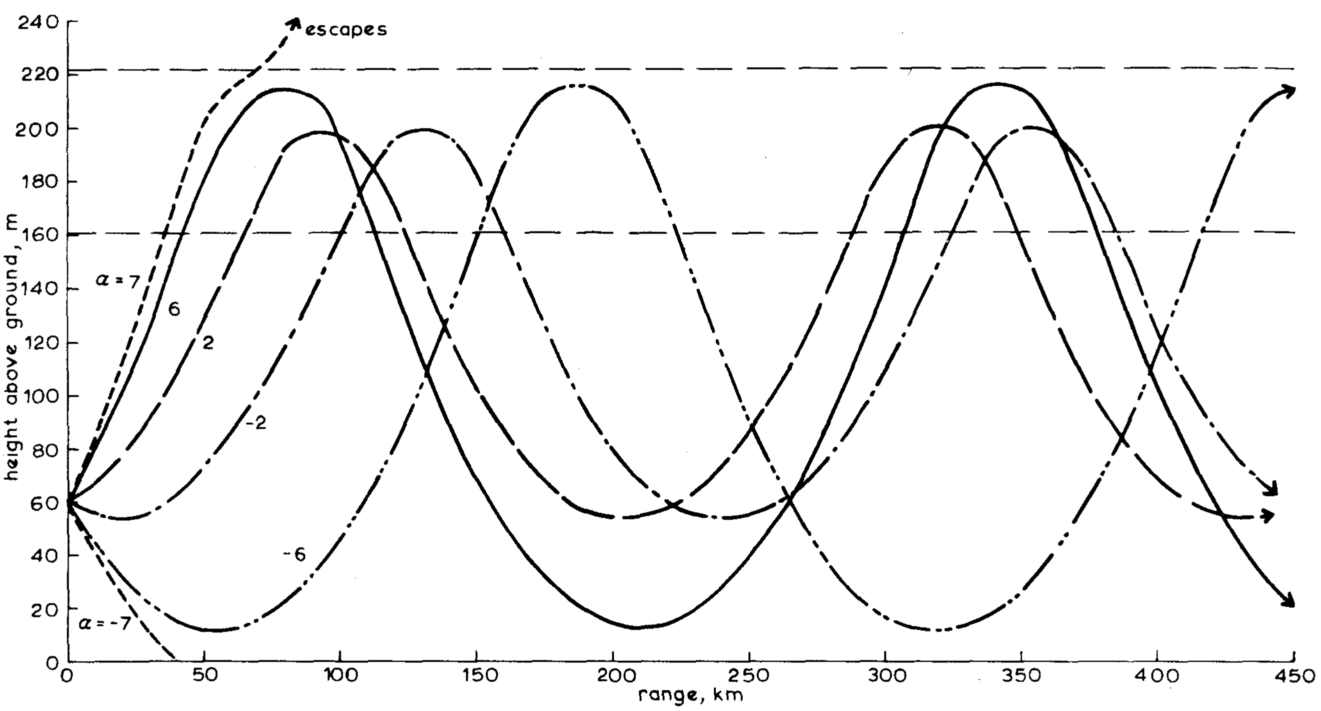

Fig. 5 - Ray set in the troposphere: illustrating ducting phenomena
α = take-off angle in minutes of arc

5. ILLUSTRATIVE RESULTS

Fig. 5 shows a ray set obtained in the tropospheric application when the refractive-index gradients of the air layers close to the ground are conducive to the formation of elevated ducts. The mechanism of long-range propagation from an elevated transmitting aerial under these conditions is clearly demonstrated. The number labelling each ray in the figure is the take-off angle in minutes of arc. The computing time (including print-out) using the Elliot 803B is approximately one minute per ray.

Fig. 6 shows the trajectories of several ordinary rays through the ionosphere with a fixed take-off angle but varying wave frequencies. An electron-density profile for the ionosphere was assumed which corresponded to average night-time conditions found in Europe. The greater penetration and range of the higher-frequency waves is evident. The asymmetry of the trajectories is due to the difference in the angle between the ray direction and Earth's magnetic field for the upward and downward paths.

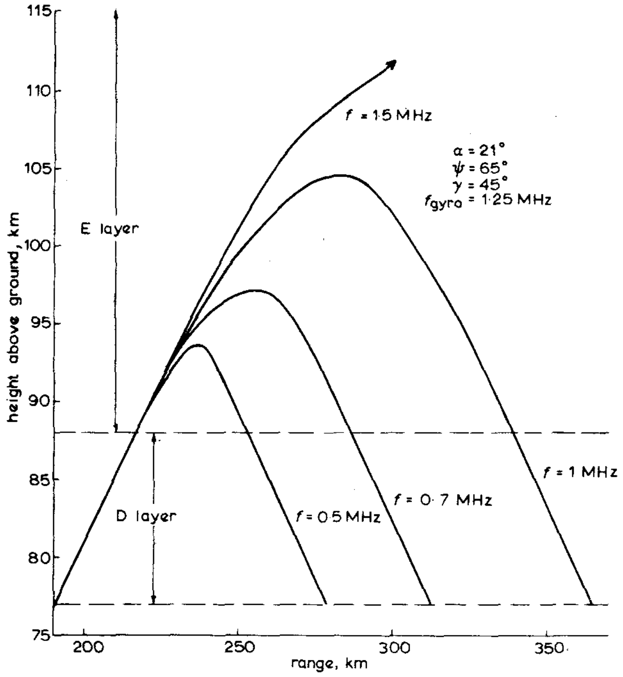

Fig. 6 - Ordinary rays in the ionosphere:
effect of wave frequency

Fig. 7 shows a plot of ray height versus cumulative absorption loss for two ordinary rays as they progress through the ionosphere. The rays have the same wave frequencies and are launched by a transmitter in an equatorial location with the same take-off angle, and the ground ranges are approximately equal. The solid-line upper curve refers to east-west propagation while the dashed-line curve refers to north-south propagation. The lower absorption loss for north-south propagation in in equatorial regions is clearly indicated.

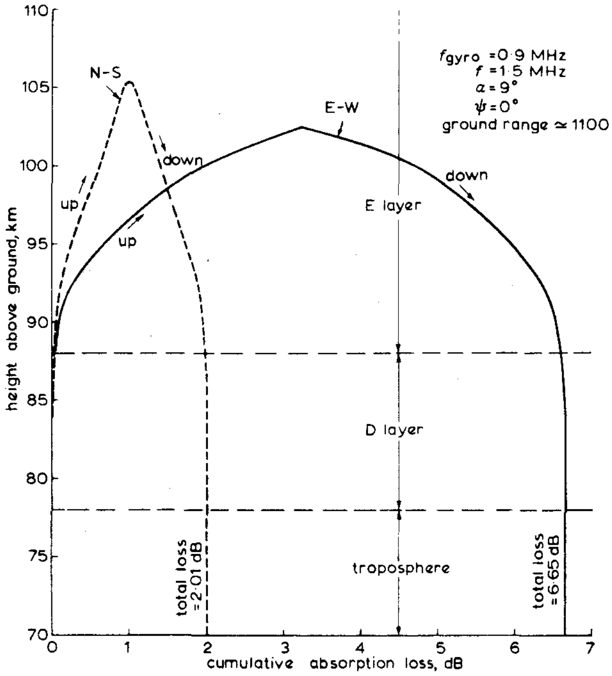

Fig. 7 - Ionospheric absorption losses (ordinary waves): variation with height

——— East-west propagation at the equator
- - - - - North-south propagation at the equator

6. CONCLUSIONS

A simple computer programme for conventional ray tracing, such as that outlined in this report, is certainly a useful tool for exploratory research into several aspects of propagation phenomena. It is attractive because of the rapidity with which results can be obtained, the control of the parameters which the investigator can exercise, and the potential adaptability to media with a wide range of characteristics. Extension of the present programme to include other more complicated forms of layer stratification should not prove difficult.

It must be emphasized that, even for those conditions where a geometrical ray-theory is a valid approximation to a full wave treatment, the realism of the results obtained is governed largely by the accuracy with which the physical characteristics of the stratified media can be ascertained by measurement.

7. REFERENCES

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APPENDIX

Calculation of Refractive-Index Gradient

Normally, the index gradient is most easily calculated from the successively computed values of μ. Thus the differential coefficient $(\mathrm{d}\mu/\mathrm{d}p)_k$ required in the trajectory equation is obtained by calculating $(\mu_k-\mu_{k-1})/(p_k-p_{k-1})$. At the top of the trajectory, however, it may happen that p_k becomes very close or even equal to p_{k-1} and the gradient calculated in this manner is likely to be in error, leading to distortion of the ray trajectory and errors in the absorption loss and ground range. In this event, i.e. when $p_k \to p_{k-1}$, it is better to calculate the gradient from the derivative of the Appleton-Hartree formula, with respect to the relative height p. As a first approximation one may use the partial derivative $(\partial\mu/\partial p)_k$. The approximation will be good if the rate of change of the ray direction (which, unfortunately, is a maximum at the turn-over point) does not significantly affect the index gradient.

In order to simplify the calculation, it is assumed that the collision frequency does not affect the ray path near the turn-over point so that Z can be set equal to zero in the Appleton-Hartree formula.

Thus from

$$\mu^2 = 1 - X\left\{1 - \frac{Y^2\sin^2\theta}{2\,(1-X)} \pm \left[\frac{Y^4\sin^4\theta}{4\,(1-X)^2} + Y^2\cos^2\theta\right]^{\frac{1}{2}}\right\}^{-1}$$

one obtains

$$\frac{\partial\mu}{\partial p} = \frac{-(1-\mu^2)}{2\mu X}\left\{\frac{U(1-\mu^2)}{1-X}\left[\frac{X-(1-\mu^2)}{X+(U-1)(1-\mu^2)}\right]+1\right\}\frac{\mathrm{d}X}{\mathrm{d}p}$$

where,

$$U = \frac{Y^2\sin^2\theta}{2(1-X)}$$

By definition $X = 81E/f^2$, therefore

$$\frac{\mathrm{d}X}{\mathrm{d}p} = \frac{X}{E}\cdot\frac{\mathrm{d}E}{\mathrm{d}p}$$

Hence substituting and inserting subscripts the partial derivative is:

$$\left(\frac{\partial\mu}{\partial p}\right)_k = \frac{-(1-\mu_k^2)}{2\mu_k\,E_k}\left\{\frac{U_k(1-\mu_k^2)}{1-X_k}\left[\frac{X_k-(1-\mu_k^2)}{X_k+(U_k-1)\,(1-\mu_k^2)}\right]+1\right\}\left(\frac{\mathrm{d}E}{\mathrm{d}p}\right)_k$$

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